

## pg 66 Maintaining Mathematical Proficiency

$$40. \begin{array}{r} x+3y=6 \\ -x-2y=-5 \\ \hline y=1 \\ (3,1) \end{array} \quad \begin{array}{r} x+3(1)=6 \\ =3-3 \\ \hline x=3 \end{array}$$

$$41. \begin{array}{r} 2x-y=-3 \\ -5x+y=3 \\ \hline -3x=0 \\ =3 \quad 3 \\ \hline x=0 \end{array}$$

$$\begin{array}{r} 5(0)+y=3 \\ \hline y=3 \\ (0,3) \end{array}$$

$$42. \begin{array}{r} 4x+2y=-4 \rightarrow 4x+2y=-4 \\ 2(-2x+6y=44) \rightarrow -4x+12y=88 \\ \hline 14y=84 \\ y=6 \\ 4x+2(6)=-4 \\ 4x+12=-4 \\ \hline 4x=-16 \\ x=-4 \\ (-4,6) \end{array}$$

$$43. \begin{array}{r} (4x-3y=9) \cdot 7 \rightarrow -28x+21y=-63 \\ 5x-21y=6 \rightarrow 5x-21y=6 \\ \hline -23x=-69 \\ =23 \quad -23 \\ \hline x=3 \\ 4(3)-3y=9 \\ 12-3y=9 \\ \hline -3y=-3 \\ y=1 \\ (3,1) \end{array}$$

## 2.2 Solving Linear Systems by Elimination

- In each method, we eliminate a variable to get a system of two variables.
- If there are infinitely many solutions when the Rouché's C.C.

$$\begin{aligned} 3. \quad & x + y - 2z = 5 \\ & -x + 2y + z = 2 \\ & 2x + 3y - z = 9 \end{aligned}$$

$$\begin{aligned} & x + y - 2z = 5 \\ & -x + 2y + z = 2 \\ \hline & 3y - z = 7 \end{aligned}$$

$$\begin{aligned} & -2x + 4y + 2z = 4 \\ & 2x + 3y - z = 9 \\ \hline & 7y + z = 13 \\ & 7(2) + z = 13 \\ & 14 + z = 13 \\ & z = -1 \end{aligned}$$

$$\begin{aligned} & 3y - z = 7 \\ & 7y + z = 13 \\ \hline & 10y = 20 \\ & 10 \quad 10 \\ \hline & y = 2 \end{aligned}$$

$$\begin{aligned} & x + (2) - 2(-1) = 5 \\ & x + 2 + 2 = 5 \\ & x + 4 = 5 \\ & x = 1 \end{aligned}$$

$$\boxed{(1, 2, -1)}$$

$$\boxed{z = -1}$$

$$\begin{aligned} 4. \quad & x + 4y - 6z = 1 \\ & 2x - y + 2z = -7 \\ & -x + 2y - 4z = 5 \end{aligned}$$

$$\begin{aligned} & x + 4y - 6z = 1 \\ & -x + 2y - 4z = 5 \\ \hline & 6y - 10z = 4 \end{aligned}$$

$$\begin{aligned} & 2x - y + 2z = -1 \\ & -2x + 4y - 8z = 10 \\ \hline & 3y - 6z = 3 \\ & 3y - 6(-1) = 3 \\ & 3y + 6 = 3 \\ & 3y = -3 \\ & y = -1 \end{aligned}$$

$$\begin{aligned} & 6y - 10z = 4 \\ & -6y + 12z = -6 \\ \hline & 2z = -2 \\ & z = -1 \end{aligned}$$

$$\begin{aligned} & x + 4(-1) - 6(-1) = 1 \\ & x - 4 + 6 = 1 \\ & x + 2 = 1 \\ & x = -1 \end{aligned}$$

$$\boxed{(-3, -1, -1)}$$

$$\boxed{y = -1}$$

$$\begin{aligned} 5. \quad & 3x - y + 2z = 4 \\ & 6x - 2y + 4z = -8 \\ & 2x - y + 3z = 10 \end{aligned}$$

$$\begin{aligned} & -6x + 2y - 4z = -8 \\ & 6x - 2y + 4z = -8 \\ \hline & 0 = -16 \end{aligned}$$

$\boxed{\text{NO SOLUTION}}$

$$6. \quad \begin{aligned} & 5x + y - z = 6 \\ & x + y + z = 2 \end{aligned}$$

$$\begin{aligned} & 5x + y - z = 6 \\ & x + y + z = 2 \end{aligned}$$

$$\begin{aligned} & -12y - 4z = -16 \\ & 12y + 4z = 16 \end{aligned}$$

$\boxed{\text{NO SOLUTION}}$

$$\begin{aligned} 8. \quad & 2x - 3y + z = -6 \\ & x + y + z = 5 \\ & -x + 2y - 6z = 31 \end{aligned}$$

$$\begin{aligned} & -2x - 3y + z = -6 \\ & x + y - z = 5 \\ & -x - 2y = -1 \end{aligned}$$

$$\begin{aligned} & -12x - 18y + z = 26 \\ & 7x + 13y - 4z = 31 \\ & -5x - 10y = -5 \\ & 5x + 10y = 5 \\ & 0 = 0 \end{aligned}$$

Infinite Solutions

9. Did not multiply the constant (11) by 4

10. Did not multiply the constant (-18) by 3

$$\begin{aligned} 11. \quad & x + y - z = 4 \\ & 3x + 2y + 4z = 17 \\ & -x + 5y + z = 8 \end{aligned}$$

$$\begin{aligned} & x + y - z = 4 \\ & -x + 5y + z = 8 \\ \hline & 6y = 12 \\ & y = 2 \end{aligned}$$

$$\begin{aligned} & x + 2 - z = 4 \\ & x - z = 2 \end{aligned}$$

$$\begin{aligned} & 3x + 2(2) + 4z = 17 \\ & 3x + 4z = 13 \\ & -3x + 3z = -6 \\ \hline & 7z = 7 \\ & z = 1 \end{aligned}$$

$$x + 2 - 1 = 4$$

$$\begin{aligned} x + 1 &= 4 \\ x &= 3 \end{aligned}$$

(3, 2, 1)

$$\begin{aligned} 12. \quad & 2x - y - z = 15 \\ & 4x + 5y + 2z = 10 \\ & -x - 4y + 3z = -20 \end{aligned}$$

$$\begin{aligned} & -4x + 2y + 2z = -30 \\ & 4x + 5y + 2z = 10 \\ \hline & 7y + 4z = -20 \end{aligned}$$

$$\begin{aligned} & 4x + 5y + 2z = 10 \\ & -4x - 10y + 2z = -80 \\ \hline & -11y + 4z = -70 \end{aligned}$$

$$\begin{aligned} 7y + 4z &= -20 \rightarrow \\ -11y + 4z &= -70 \rightarrow \end{aligned}$$

$$\begin{aligned} & -49y - 28z = 140 \\ & -22y + 28z = -140 \\ \hline & -71y = 0 \\ & y = 0 \end{aligned}$$

$$\begin{aligned} 7y + 4z &= -20 \\ 2(0) + 4z &= -20 \\ \hline 4z &= -20 \\ z &= -5 \end{aligned}$$

$$\begin{aligned} 2x - 0 + (-5) &= 15 \\ 2x - 5 &= 15 \\ 2x &= 20 \\ x &= 10 \end{aligned}$$

(10, 0, -5)

$$\begin{aligned} 13. \quad & x + 2y - z = 3 \\ & 2x + 4y - 2z = 6 \\ & -x - 2y + z = -6 \end{aligned}$$

$$\begin{aligned} & x + 2y - z = 3 \\ & -x - 2y + z = -6 \\ \hline & 0 = -3 \end{aligned}$$

No solution

$$\begin{aligned} 14. \quad & x + 2y + 3z = 4 \\ & -3x + 2y - z = 12 \\ & -2x - 2y - 4z = -14 \end{aligned}$$

$$\begin{aligned} & -x - 2y - 3z = -4 \\ & -3x + 2y - z = 12 \\ \hline & -4x - 4z = 8 \end{aligned}$$

$$\begin{aligned} & -3x + 2y - z = 12 \\ & -2x - 2y - 4z = -14 \\ \hline & -5x - 5z = -2 \end{aligned}$$

$$\begin{aligned} & 20x + 20z = -40 \\ & -20x - 20z = -8 \\ \hline & 0 = -48 \end{aligned}$$

No solution

$$\begin{aligned} 15. \quad & x+2y-z=3 \\ & -2x-y+z=-1 \\ & 6x-3y-z=-7 \end{aligned}$$

$$\begin{aligned} & x+2y-z=3 \\ & -2x-y+z=-1 \\ & -x+y-z=2 \end{aligned}$$

$$\begin{aligned} & -6x-3y+z=-7 \\ & -2x-y+z=-1 \\ & 4x+2y-z=-8 \\ & -4x+4y=8 \\ & 0=0 \end{aligned}$$

Infinite Solutions

$$\begin{aligned} 16. \quad & 4x+y+5z=5 \\ & 2x+2y+10z=11 \\ & x-y+2z=-2 \end{aligned}$$

$$\begin{aligned} & -2x-2y-10z=10 \\ & 2x+2y+10z=11 \\ & 0=0 \end{aligned}$$

Infinite Solutions

$$\begin{aligned} 17. \quad & 2p+l+s=14 \\ & 1p+l+3s=15 \\ & 3p+l+7s=22 \end{aligned}$$

$$\begin{aligned} & 2p+l+s=14 \\ & -p-l-3s=15 \\ & p-7s=-1 \end{aligned}$$

$$\begin{aligned} & -p-l-3s=15 \\ & 3p+l+7s=22 \\ & 2p-s=7 \\ & -4p+4s=2 \\ & 2s=9 \\ & s=3 \end{aligned}$$

$$\begin{aligned} & 2p-7s=14 \\ & -p-l-3s=15 \\ & 2p-7s=14 \\ & p=5 \end{aligned}$$

$$\begin{aligned} & 2(5)+l+3=14 \\ & 13+l=14 \\ & -l=-1 \\ & l=1 \end{aligned}$$

Pizza \$5  
Soda \$1  
Salad \$3

$$\begin{aligned} 18. \quad & s+l=1300 \\ & s+2c=1400 \\ & s+l+c=1600 \end{aligned}$$

$$\begin{bmatrix} 1 & 1 & 0 & 1300 \\ 1 & 0 & 2 & 1400 \\ 1 & 1 & 1 & 1600 \end{bmatrix}$$

$R_1 + R_2$

$$\begin{bmatrix} 1 & 1 & 0 & 1300 \\ 0 & -1 & 2 & 100 \\ 1 & 1 & 1 & 1600 \end{bmatrix}$$

$R_1 + R_3$

$$\begin{bmatrix} 1 & 1 & 0 & 1300 \\ 0 & -1 & 2 & 100 \\ 0 & 0 & 1 & 300 \end{bmatrix}$$

$$\begin{aligned} & s+l=1300 \\ & -l+2c=100 \\ & c=300 \end{aligned}$$

$$\begin{aligned} & -l+2(300)=100 \\ & -l+600=100 \\ & -600-600 \\ & l=1500 \end{aligned}$$

$$\begin{aligned} & s+500=1300 \\ & -500-500 \\ & s=800 \end{aligned}$$

Sofa \$800  
Love seat \$500  
Chair \$300

$$\begin{aligned} 19. \quad & a+s+c=400 \\ & 22a+15s+13.50c=7840 \\ & c=s+40 \end{aligned}$$

$$\begin{aligned} & a+s+(s+40)=400 \\ & -40-40 \\ & a+2s=360 \\ & a=360-2s \end{aligned}$$

$$\begin{aligned} & 22a+15s+13.50(s+40)=7840 \\ & 22a+15s+13.50s+540=7840 \\ & -840-540 \end{aligned}$$

$$\begin{aligned} & c=40+40 \\ & c=80 \end{aligned}$$

$$\begin{aligned} & 22a+28.50s=7300 \\ & 22(360-2s)+28.50s=7300 \\ & 7920-44s+28.50s=7300 \\ & -7920 \end{aligned}$$

$$\begin{aligned} & a+40+80=400 \\ & a+120=400 \\ & -120-120 \\ & a=280 \end{aligned}$$

280 Adults  
40 Students  
80 Children

$$\begin{aligned} & -15.5s=-620 \\ & 15.5s=620 \\ & s=40 \end{aligned}$$

$$\begin{aligned}
 20. \quad & b + l + u = 10,000 \\
 & 10b + 8l + 5u = 70,000 \\
 & u = 4b \\
 & \boxed{u = 4000}
 \end{aligned}$$

**5000 lower deck**

$$\begin{aligned}
 & b + l + 4u = 10,000 \\
 & 5b + l = 10,000 \\
 & l = -5b + 10,000 \\
 & l = 5(1000) + 10,000 \\
 & l = 5000 + 10,000 \\
 & \boxed{l = 5000}
 \end{aligned}$$

$$\begin{aligned}
 & 10b + 8l + 5(4b) = 70,000 \\
 & 10b + 8l + 20b = 70,000 \\
 & 30b + 8l = 70,000 \\
 & 30b + 8(-5b + 10,000) = 70,000 \\
 & 30b - 40b + 80,000 = 70,000 \\
 & -10b = -10,000 \\
 & \frac{-10b}{-10} = \frac{-10,000}{-10} \\
 & \boxed{b = 1000}
 \end{aligned}$$

21. a) elimination - the x's can easily be eliminated  
 b) Gaussian → divide equation 3 by 2 so the coefficient on x is one

$$\begin{aligned}
 22. \quad & q + 2d + n = .50 \\
 & q + 2d + 3n = .60 \\
 & 2q + d + n = .65
 \end{aligned}$$

$$\begin{aligned}
 23. \quad & -1 + 2(2) - 3(-3) = a \\
 & -1 + 4 + 9 = a \\
 & 3 + 9 = a \\
 & \boxed{12 = a}
 \end{aligned}$$

$$\begin{aligned}
 & -(-1) - 2 + (-3) = b \\
 & 1 - 2 - 3 = b \\
 & -1 - 3 = b \\
 & \boxed{-4 = b}
 \end{aligned}$$

$$\begin{aligned}
 & 2(-1) + 3(2) - 2(-3) = c \\
 & -2 + 6 + 6 = c \\
 & 4 + 6 = c \\
 & \boxed{10 = c}
 \end{aligned}$$

$$\begin{aligned}
 24. \quad & 4x + y + z = 0 \\
 & 2x + 12y - 3z = 0 \\
 & -x - 14y - z = 0
 \end{aligned}$$

$$\begin{aligned}
 & 4x + y + z = 0 \\
 & -4x - 14y + 3z = 0 \\
 & 7z = 0 \\
 & z = 0
 \end{aligned}$$

$$\begin{aligned}
 & 4x + y + z = 0 \\
 & -4x - 14y - 4z = 0 \\
 & -3z = 0 \\
 & z = 0
 \end{aligned}$$

**yes, the x & y values will cancel every time**

$$\begin{aligned}
 25. \quad & a) \quad a = -1 \quad b = -1 \quad c = -1 \\
 & x + y + z = 2 \\
 & -x - y - z = 10 \\
 & 0 = 12
 \end{aligned}$$

$$\begin{aligned}
 & b) \quad a = 3 \quad b = 2 \quad c = 4 \\
 & 3x + 2y + 4z = 10 \\
 & x - 2y + z = 4 \\
 & \hline
 & 4x + 5z = 14
 \end{aligned}$$

can use elimination

$$\begin{aligned}
 & c) \quad a = 5 \quad b = 5 \quad c = 5 \\
 & x + y + z = 2 \rightarrow -5x + -5y - 5z = -10 \\
 & 5x + 5y + 5z = 10 \\
 & 0 = 0
 \end{aligned}$$

$$\begin{aligned}
 26. \quad & g + r + e = 21 \\
 & 1.25g + r + 1.5e = 24 \\
 & \boxed{g = 2} \\
 & \rightarrow -r + e = 19 \\
 & \rightarrow 1 + 1.5e = 21.5 \\
 & 1.5e = 20.5 \\
 & \cdot \frac{2}{3} \quad \cdot \frac{2}{3} \\
 & \boxed{e = 5}
 \end{aligned}$$

$$\begin{aligned}
 & 2 + r + 5 = 21 \\
 & 7 + r = 21 \\
 & -7 \quad -7 \\
 & \boxed{r = 14}
 \end{aligned}$$

**2 lbs golden  
 14 lbs red  
 5 lbs empire**